

**Exercise:** Consider the control problem,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

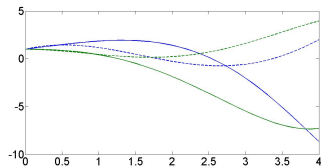
$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

1. Show that the Kalman rank condition is satisfied and the system is controllable.
2. Solve the system numerically (use Euler, trapezoidal or a Runge Kutta method) without control, i.e.  $u = 0$ .
3. Compute a control  $u(t)$  for which the solution satisfies

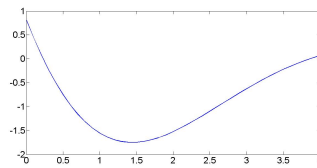
$$y_1(4) = 2, \quad y_2(4) = 4.$$

Draw the control  $u$ .

4. Draw both the uncontrolled solution and the controlled one in the same figure.



solutions with and without control



control